International Journal of Engineering, Science and Mathematics
Vol. 8 Issue 1, January 2019,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

The Concept of Weakly Regular Rings

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ABSTRACT

The concept of weakly regular rings were exclusively studied by Kovacs, Brown & McCoy, Ramamurthy, Camilo & Xiao & others, generalizes a property of regular rings. Characterization of weakly regular ring by Ramamurthy has found in theorem we find that a regular ring in right weakly regular. The theorem is a generalized properties of a right weakly regular rings. A ring which is both-left and right weakly regular is called weakly regular. For right artinian rings, right weak regularity is equivalent to regularity and biregularity.

INTRODUCTION :-

WEAKLY REGULAR RINGS:-

The notion of weakly regular rings was introduced by Ramamurthy and significant result have been obtained on weakly regular rings by Camillo and Xiao. Ramamurthy generalize a property of regular ring, namely, $l^2 = l$ for each right (left) ideal. A ring which is both-left and right weakly regular is called weakly regular. Kovacs proved that, for commutative rings, weak regularity and regularity are equivalent conditions for arbitrary rings. In general weak regularity don't imply regularity. In fact, the class of weakly regular rings strictly contains the class ofregular rings as well as class of biregular rings. For right artinian rings, right weak regularity is equivalent to regularity.

1.1 **DEFINITION:** Aring is said to be right (left) weakly regular if $l^2=l$ for each ring (left) ideal I in R. R will be said to be weakly regular if it is both right and left weakly regular.

A ring R is right (left) D-regular if $x \in x R$ ($x \in R x$) for each $x \in R$. A ring R is D-regular if it is both left and right D-regular.

The following characterization for right weakly regular rings have been obtained by Ramamurthy.

THEOREM: The following conditions are equivalent for any ring R:

(i) R is right weakly regular.

(ii) For every $x \in R$, $r \in (rR)^2$

(iii) For any two right ideal l_1 and l_2 in R.

$$l_1 \subseteq l_2 \Longrightarrow l_1 l_2 = l_1$$

PROOF: Suppose (i) holds, let I be the right ideal generated by $r \in R$. then we see that.

$$l^2 \subseteq r R \subseteq l$$

Also $\quad r \in l = l^2 \subseteq r R \subseteq l$

Therefore, $\quad l = r R$ and hence

$$r \in l^2 = (rR)^2$$

Thus, (i) $\Longrightarrow$ (ii)

Next, suppose (ii) holds

Let $r \in l$

Then $r \in (rR)^2$ by condition (ii)

But $r \in l$, and l_1 being a right ideal

$rR \subseteq l_1 \subseteq l_2$.

Therefore $(rR)^2 \subseteq l_1\, l_2$

Hence $l_1 \subseteq l_1\, l_2$

The reverse inequality $l_1\, l_2 \subseteq l$ is true for any two right ideal l_1 and l_2

Therefore, $l_1\, l_2 = l$

Thus proves that (ii) ⟹ (iii)

Finally, we shall prove that (iii) ⟹ (i)

For this let (iii) holds

Putting $l_1 = l_2 = l$ in (iii), we get $l^2 = l$, for each right ideal l in R.

Hence R is right weakly regular,

Proving (iii) ⟹ (i)

REMARK:

(a) It has been proved above that in a right weakly regular ring R, the right principal ideal generated by an element r of R is rR. This result also holds in regular ring and strongly regular ring.

(b) The condition $r \in (rR)^2$ for each $x \in R$ are equivalently be replaced by $r \in (rR)^{\mathbf{n}}$, for some $n \geq 2$ and each $r \in R$.

(c) It also follows from the above theorem that a regular rings is right weakly regular.

(d) A theorem with right replaced by the let in the theorem [1.2] will also hold in the following form:

The following conditions are equivalent for a ring R.

(i) R is left weakly regular

(ii) $r \in (Rr)^2$, for each $r \in R$.

(iii) for any two left ideal 1_l and l_2 of R. $l_1 \subseteq l_2 = l_1\, l_2 = 1_2$.

The following theorem has been proved by Ramamurthy.

<u>This topic are concerning theorem:</u>

<u>THEOREM: I</u>

The following conditions on a ring R are equivalent:

i. R is a weekly regular ring.

ii. For each element $a \in Re$ where e is a non unit idempotent element of R then a $\in Ra^2$.

iii. For each element $a \in Re$ where e is a non unit idempotent element of R then R is a direct summand of R.

<u>THEOREM: II</u>

If R be a ring then the following conditions are equivalent:

i. R is a regular ring.

ii. R is a weekly regular ring and for every non unit element of R there exists an idempotent $e \neq 1$ of R such that $a \in Ra$.

Now we give a necessary and sufficient condition for a direct product of rings to be a weakly regular ring.

<u>THEOREM: III</u>

If R be a ring I be a primary ideal then R/l is a weekly regular ring.

<u>THEOREM: IV</u>

If R be a ring then the polynomial ring R is a weekly regular ring of and only if R has exactly two idempotent elements.

PROOF: If R is a ring in which the only idempotent elements are 0 and 1 then R is weekly regular ring. In particular if R be an integral domain or a local ring then R is a weekly; regular ring.

To prove theorem, suppose R has exactly two idempotent elements. Since the set of all idempotent element of polynomial ring R is $\{a \in R;\ a^2=a\}$, then R is weekly regular ring.

Conversely let R[x] is a weekly regular ring and e be a non unit idempotent element of R.

We have $ex \in e\,R[x]$

Now using the condition (ii) of the theorem

We get $ex \in (ex)^2 R[x]$

Then there exist some element $f \in R\,[x]$ such that $ex = ex^2 f(x)$

Thus e=0, which completes the proof of the theorem. By induction on form the above theorem we have the following corollary.

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